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Gödel, Penrose, anti-Mach: extra supersymmetries of time-dependent plane waves

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Abstract: We prove that M-theory plane waves with extra supersymmetries are necessarily homogeneous (but possibly time-dependent), and we show by explicit construction that such time-dependent plane waves can admit extra supersymmetries. To that end we study the Penrose limits of Gödel-like metrics, show that the Penrose limit of the M-theory Gödel metric (with 20 supercharges) is generically a time-dependent homogeneous plane wave of the anti-Mach type, and display the four extra Killings spinors in that case. We conclude with some general remarks on the Killing spinor equations for homogeneous plane waves.

Keywords: M-Theory, Penrose limit and pp-wave background.

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1. Introduction

The results of [1]–[4] have led to a renewed interest in various aspects of supergravity and string theory on plane wave backgrounds

$$ds^2 = 2dudv + A_{ij}(u)x^i x^j du^2 + d\vec{x}^2 \,. \tag{1.1}$$

The main focus has been on time-independent plane waves, i.e. metrics with A_{ij} constant. For example, while any plane wave solution of supergravity preserves half of the supersymmetries [5], these time-independent plane waves have been shown to realise various exotic $> 1/2$ fractions of unbroken supersymmetries [6]–[9].

In another development [10] attention was drawn to *homogeneous plane waves*. These generalise the time-independent (and symmetric homogeneous) plane waves to time-dependent plane waves in a way which does not destroy the homogeneity of the metric. These homogeneous plane waves possess a number of interesting features and string theory in these backgrounds has been studied in [11, 12].

Here we draw together these two, apparently unrelated, developments by showing

- that the existence of extra Killing spinors implies that the plane wave metric is that of a smooth homogeneous plane wave (this corresponds to one of the two families of homogeneous plane waves found in [10]),
- and (perhaps surprisingly) that such time-dependent plane waves can also have extra Killing spinors and realise exotic fractions of supersymmetries.

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The argument for the first claim is extremely simple and follows from looking at the Killing vector constructed from the extra Killing spinor (section 3.1). To establish the second claim, in section 2 we study the Penrose [13] limits of various Gödel-like metrics and show that they give rise to non-trivial homogeneous plane waves of the anti-Mach kind [14] studied in [10, 12]. In particular, the Penrose-Güven [15] limit of the M-theory Gödel metric discovered in [16], which has 20 supersymmetries, is a time-dependent anti-Mach wave solution of M-theory (section 2.3). Since the number of supersymmetries cannot decrease in the Penrose limit [17], it must therefore be true that this time-dependent plane wave admits extra Killing spinors, and we go on to construct these Killing spinors explicitly (section 3.3).[1] We also show that the existence of one extra Killing spinor implies the equations of motion (section 3.2), and we conclude with some general remarks on the Killing spinor equations for homogeneous plane waves.

2. Gödel, Penrose, anti–Mach

2.1 A two-parameter family of Gödel metrics

We first consider the two-parameter family of Gödel metrics (see e.g. [20] and references therein)

$$ds^2 = -\left(dt + \frac{4\sqrt{2}\Omega}{m^2}\sinh^2\frac{m\rho}{2}d\phi\right)^2 + d\rho^2 + \frac{1}{m^2}\sinh^2 m\rho\, d\phi^2 + dz^2\,. \tag{2.1}$$

In terms of

$$r = \sinh\frac{m\rho}{2} \tag{2.2}$$

and the parameters Ω and

$$\Delta = \frac{4\Omega^2}{m^2} \tag{2.3}$$

this family can alternatively be written as

$$ds^2 = -dt^2 - 2\sqrt{2}\frac{\Delta}{\Omega}r^2 dt d\phi + \frac{\Delta}{\Omega^2}\frac{dr^2}{1+r^2} + \frac{\Delta}{\Omega^2}(r^2 + (1-2\Delta)r^4)d\phi^2 + dz^2\,. \tag{2.4}$$

These metrics are homogeneous. The orbits of the Killing vector ∂_ϕ are closed, and since the norm of ∂_ϕ is proportional to $(1+(1-2\Delta)r^2)$, one sees that there are closed timelike curves for

$$r^2 > \frac{1}{2\Delta - 1}\,. \tag{2.5}$$

Thus these metrics share all the characteristics of the standard one-parameter family of Gödel metrics [21] (with rotation parameter Ω) which one obtains for $\Delta = 1$. The one-parameter family $\Omega^2 = \Delta$ was shown in [22] to interpolate between the Gödel metric at $\Delta = 1$ and the $AdS_3 \times \mathbb{R}$ metric at $\Delta = 1/2$ (where closed timelike curves are pushed to infinity and cease to exist).

[1] This evidently contradicts the "proof" in [18, 19] that such solutions cannot exist. This proof rests on the invalid assumption that, as in the time-independent case [7], also in the time-dependent case everything can be conjugated into the Cartan of $SO(16)$ in a time-independent way.

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Another interesting limit is $m \to 0$ with Ω fixed. Alternatively, in terms of the metric (2.4) one scales

$$r^2 \to \frac{\Delta}{\Omega^2} r^2 \tag{2.6}$$

and then takes the limit $\Delta \to \infty$. Then one finds the one-parameter family of metrics

$$\begin{aligned} ds^2 &= -dt^2 - 2\beta r^2 dt d\phi + dr^2 + (r^2 - \beta^2 r^4) d\phi^2 + dz^2 \\ &= -(dt + \beta r^2 d\phi)^2 + dr^2 + r^2 d\phi^2 + dz^2 \end{aligned} \tag{2.7}$$

($\beta = \sqrt{2}\Omega$) which is of the typical form of Gödel-like metrics encountered in string theory and M-theory [16, 23]. We will refer to it below as a stringy Gödel metric.

2.2 The Penrose limit of the stringy four-dimensional Gödel metric

We now consider the Penrose limits of the above metrics. Before embarking on the calculation let us try to anticipate what sort of result we expect. As mentioned above, the Gödel metrics are homogeneous. Now it was pointed out in [24] (by way of example) that the Penrose limit of a homogeneous space-time need not be homogeneous. However, it appears to be possible to show [25] that the Penrose limit of a homogeneous (naturally) reductive space is itself homogeneous reductive. It is not difficult to see that the Gödel metrics are actually homogeneous reductive (while the homogeneous Kaigorodov spaces considered in [24] are not). Thus we expect the Penrose limit of these Gödel metrics to be homogeneous plane waves. This expectation will be borne out by our explicit calculations below.

As it will turn out that the Penrose limits of all of the above metrics can also be obtained by starting from the simpler metric (2.7) with $\beta = 1$ (see appendix A.2), we will just look at null geodesics in this case.

The obvious Killing vectors $\partial_z, \partial_t, \partial_\phi$ of (2.7) give rise to the first integrals

$$\begin{aligned} \dot{z} &= P \\ \dot{t} &= 1 - r^2 \\ \dot{\phi} &= 1\,, \end{aligned} \tag{2.8}$$

where an overdot denotes differentiation with respect to the affine parameter τ. Without loss of generality we have set the angular momentum $p_\phi = L$ to zero. Then, using the remaining Killing vectors (appendix A.1) it is not difficult to show (and in any case straightforward to verify) that the solution for $r(\tau)$ can be chosen to be

$$r(\tau) = (1 - P^2)^{1/2} \sin\tau\,. \tag{2.9}$$

To take the Penrose limit along any of the above geodesics (parametrised by P), we change coordinates from $x^\mu = (r, z, \phi, t)$ to adapted coordinates [13, 15, 17] (u, v, y_1, y_2) with $u = \tau$. A possible choice is

$$\begin{aligned} r &= r(u) \\ dz &= dy_1 + P du \\ d\phi &= dy_2 + du \\ dt &= -dv + P dy_1 + (1 - r(u)^2) du\,. \end{aligned} \tag{2.10}$$

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Then, following the standard procedure, one finds the Penrose limit

$$\begin{aligned} ds^2 &= 2dudv + (1-P^2)dy_1^2 - 2Pr(u)^2 dy_1 dy_2 + (r(u)^2 - r(u)^4)dy_2^2 \\ r(u)^2 &= (1-P^2)\sin^2 u\,. \end{aligned} \tag{2.11}$$

The same one-parameter family of plane wave metrics is also obtained from the Penrose limits of the general two-parameter family of Gödel metrics (2.4).

As usual, the metric in Rosen coordinates is not particularly revealing. In particular, it is not obvious at this point that this is really a homogeneous plane wave. To exhibit this, we now show that we can put the above metric into the general form of a smooth homogeneous plane wave in stationary coordinates, namely [10]

$$ds^2 = 2dudv + A_{ij}x^i x^j du^2 + 2f_{ij}x^i dx^j du + d\vec{x}^2 \tag{2.12}$$

with constant symmetric and anti-symmetric matrices A_{ij} and f_{ij} respectively.

We first rescale y_1 and y_2 by $(1-P^2)^{1/2}$ so that the metric takes the form (reorganising the terms)

$$ds^2 = 2dudv + (dy_1 - P\sin^2 u dy_2)^2 + \frac{1}{4}\sin^2 2u dy_2^2\,. \tag{2.13}$$

We can deal with the second term in the standard way [15, 17], introducing

$$x_2 = \frac{1}{2}y_2 \sin 2u\,, \tag{2.14}$$

and shifting v appropriately to eliminate the $dudx_2$ cross-term. This will have the net effect of generating $dx_2^2 - 4x_2^2 du^2$. Instead of y_1 we introduce the coordinate

$$x_1 = y_1 - Py_2 \sin^2 u\,, \tag{2.15}$$

so that

$$dy_1 - P\sin^2 u dy_2 = dx_1 + 2Px_2 du\,. \tag{2.16}$$

Then one finds the metric

$$ds^2 = 2dudv + 4(P^2-1)x_2^2 + 4Px_2 dx_1 du + dx_1^2 + dx_2^2\,. \tag{2.17}$$

Finally, one more shift of v to effect a "gauge transformation" of the magnetic field term puts the metric into the form

$$ds^2 = 2dudv + 4(P^2-1)x_2^2 du^2 - 2P(x_1 dx_2 - x_2 dx_1)du + dx_1^2 + dx_2^2\,. \tag{2.18}$$

This shows that the Penrose limit of the general Gödel metric is indeed a homogeneous plane wave, with

$$A_{11} = 0\,, \qquad A_{22} = 4(P^2-1)\,, \qquad f_{12} = -P\,. \tag{2.19}$$

In particular, this one-parameter family of homogeneous plane waves is precisely of the anti-Mach kind [14] discussed in [10, 12] in the sense that in stationary coordinates one of the frequencies is zero, reflecting an additional commuting isometry.

In summary: the Penrose limit of a Gödel metric is an anti-Mach homogeneous plane wave.

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2.3 The Penrose limit of the M-theory Gödel metric

The five-dimensional Gödel metric

$$ds_5^2 = -(dt + \beta(r_1^2 d\phi_1^2 + r_2^2 d\phi_2^2))^2 + dr_1^2 + r_1^2 d\phi_1^2 + dr_2^2 + r_2^2 d\phi_2^2 \,. \tag{2.20}$$

has the remarkable property of being a maximally supersymmetric solution of minimal five-dimensional supergravity (it preserves eight supercharges). And it has the equally remarkable property that its M-theory lift

$$ds_M^2 = ds_5^2 + dz^2 + \sum_{i=5}^{9} dx_i^2 \tag{2.21}$$

(we have singled out one of the new six transverse dimensions) supported by the four-form field strength

$$F_{r_1\phi_1 56} = F_{r_1\phi_1 78} = F_{r_1\phi_1 9z} = F_{r_2\phi_2 56} = F_{r_2\phi_2 78} = F_{r_2\phi_2 9z} = -2\beta \,, \tag{2.22}$$

preserves not only eight but actually 20 of the 32 supercharges of eleven-dimensional supergravity [16]. The analysis of null geodesics in this metric [20], using the isometry algebra determined in [16], as well as the subsequent Penrose limit of the metric, proceed in close analogy with the four-dimensional case discussed above, and we will be brief.

We choose the null geodesic to have momentum P along one of the six directions transverse to the five-dimensional Gödel metric which, without loss of generality, we can choose to be the z-direction. Then similar to (2.8) we have

$$\begin{aligned} \dot{z} &= P \\ \dot{t} &= 1 - (r_1^2 + r_2^2) \\ \dot{\phi}_1 &= \dot{\phi}_2 = 1 \,. \end{aligned} \tag{2.23}$$

where

$$r_1(\tau)^2 + r_2(\tau)^2 = (1 - P^2)\sin^2\tau \tag{2.24}$$

replaces (2.9). This we solve as

$$\begin{aligned} r_1(\tau, \alpha) &= (1 - P^2)^{1/2} \cos\alpha \sin\tau \\ r_2(\tau, \alpha) &= (1 - P^2)^{1/2} \sin\alpha \sin\tau \,, \end{aligned} \tag{2.25}$$

where α is the angle between r_1 and r_2 [20]. The analogous adapted coordinates are (cf. (2.10)) $(u, v, y_1, \tilde{\phi}_1, \tilde{\phi}_2, \alpha)$ defined by

$$\begin{aligned} r_i &= r_i(u, \alpha) \\ dz &= dy_1 + P du \\ d\phi &= d\tilde{\phi}_i + du \\ dt &= -dv + P dy_1 + (1 - (r_1(u)^2 + r_2(u)^2)) du \,. \end{aligned} \tag{2.26}$$

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We take the Penrose limit along a geodesic sitting at $\alpha = \alpha_0$ and hence introduce a new coordinate y_3 (y_2 will appear shortly ...) via

$$\alpha = y_3 + \alpha_0 \,. \tag{2.27}$$

Then one finds from (2.20) that the Penrose limit metric is

$$\begin{aligned} ds^2 \;=\; & 2dudv + (1-P^2)dy_1^2 + (r_1(u,\alpha_0)^2 + r_2(u,\alpha_0)^2)dy_3^2 + \\ & + (r_1(u,\alpha_0)^2 - r_1(u,\alpha_0)^4)d\tilde{\phi}_1^2 + (r_2(u,\alpha_0)^2 - r_2(u,\alpha_0)^4)d\tilde{\phi}_2^2 - \\ & - 2Pr_1(u,\alpha_0)^2 dy_1 d\tilde{\phi}_1 - 2Pr_2(u,\alpha_0)^2 dy_1 d\tilde{\phi}_2 - \\ & - 2r_1(u,\alpha_0)^2 r_2(u,\alpha_0)^2 d\tilde{\phi}_1 d\tilde{\phi}_2 + \sum_{i=5}^{9} dx_i^2 \,. \end{aligned} \tag{2.28}$$

To disentangle this (and eliminate the apparent dependence of the metric on α_0) one can introduce the coordinates

$$\begin{aligned} y_2 \;&=\; \cos^2\alpha_0\,\tilde{\phi}_1 + \sin^2\alpha_0\,\tilde{\phi}_2 \\ y_4 \;&=\; \sin\alpha_0\cos\alpha_0(\tilde{\phi}_2 - \tilde{\phi}_1) \,. \end{aligned} \tag{2.29}$$

Then the metric takes the form

$$\begin{aligned} ds^2 \;=\; & 2dudv + (1-P^2)\sin^2 u(dy_3^2 + dy_4^2) + \sum_{i=5}^{9} dx_i^2 + \\ & + (1-P^2)dy_1^2 - 2Pr(u)^2 dy_1 dy_2 + (r(u)^2 - r(u)^4)dy_2^2 \,, \end{aligned} \tag{2.30}$$

where, as in (2.11),

$$r(u)^2 = (1-P^2)\sin^2 u \,. \tag{2.31}$$

We see that the resulting metric is quite simple: it has the form of a standard symmetric (Cahen-Wallach) plane wave in the (y_3, y_4)-directions, and of the four-dimensional anti-Mach plane wave (2.11) in the (y_1, y_2)-directions (times a flat $\mathbb{R}^5$).

We go to stationary coordinates as before. First we scale all the y_i by $(1-P^2)^{1/2}$. Then we go to Brinkmann coordinates $x_{3,4}$ for the $y_{3,4}$-directions,

$$x_{3,4} = \sin u\, y_{3,4} \tag{2.32}$$

and to coordinates $x_{1,2}$ as in (2.15), (2.14),

$$\begin{aligned} x_1 \;&=\; y_1 - Py_2\sin^2 u \\ x_2 \;&=\; \frac{1}{2}y_2\sin 2u \,. \end{aligned} \tag{2.33}$$

(all this accompanied by an appropriate shift in v). Then one finds the metric

$$ds^2 = 2dudv + (4(P^2-1)x_2^2 - x_3^2 - x_4^2)du^2 - 2P(x_1dx_2 - x_2dx_1)du + \sum_{i=1}^{9} dx_i^2 \,. \tag{2.34}$$

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This is a non-trivial homogeneous plane wave with

$$\begin{aligned} A_{22} &= 4(P^2-1) \\ A_{33} &= A_{44} = -1 \\ f_{12} &= -P\,. \end{aligned} \tag{2.35}$$

As a check on this, note that for $P=0$ the null geodesic lies entirely in the five-dimensional Gödel metric, and must therefore lead to either the five-dimensional maximally supersymmetric plane wave [26] times $\mathbb{R}^6$ (which also has 20 supersymmetries [7]) or flat space. By inspection, one sees that it is the former.

It remains to determine the four-form field strength in the Penrose limit. Using Güven's prescription [15, 17] for taking the Penrose limit of supergravity fields other than the metric, and tracing through the chain of coordinate transformations required to put the resulting metric into the simple form (2.34), one finds that

$$F_4 = 2du \wedge [-dx^{129} + P dx^{349} - (1-P^2)^{1/2}(dx^{256} + dx^{278})]\,. \tag{2.36}$$

Here we used the shorthand notation

$$dx^{abc} = dx^a \wedge dx^b \wedge dx^c\,. \tag{2.37}$$

As another check one can verify that the above metric and F_4 indeed satisfy the supergravity equations of motion (3.37). Note that, even though the metric is trivial in the $(x_5,\ldots,x_9)$-plane, there is non-trivial flux in those directions. In section 3.3 we will show by explicit construction that this supergravity configuration has 20 (thus 4 extra) supersymmetries.

3. Extra killing spinors and homogeneous plane waves

3.1 Extra killing spinors ⇒ homogeneity

There is a very simple argument that shows that the existence of an extra Killing spinor implies that the plane wave is *homogeneous*. A small refinement of this argument also shows that this homogeneous plane wave must be smooth (corresponding to one of the two families of homogeneous plane waves found in [10]).

For definiteness we phrase the argument in the context of eleven-dimensional supergravity, but it is clearly more general than that. We write the general plane wave metric as

$$ds^2 = 2dudv + A_{ij}(u)x^i x^j du^2 + d\vec{x}^2\,. \tag{3.1}$$

In a frame basis this metric is

$$ds^2 = 2e^+e^- + (e^i)^2\,, \tag{3.2}$$

where

$$\begin{aligned} e^+ &= dv + \frac{1}{2}A_{ij}(u)x^i x^j du \\ e^- &= du \\ e^i &= dx^i\,. \end{aligned} \tag{3.3}$$

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It is well known [5] that any plane wave has 16 standard supersymmetries, corresponding to Killing spinors ϵ with $\Gamma^-\epsilon = \Gamma_+\epsilon = 0$. Extra Killing spinors are thus characterised by the condition

$$\Gamma^-\epsilon \neq 0\,. \tag{3.4}$$

Following the conventions of [7] we will also adopt in the following, we choose

$$\Gamma_\pm = \mathbb{I}_{16} \otimes \sigma_\pm \tag{3.5}$$

with $\sigma_\pm = (\sigma_1 \pm i\sigma_2)/\sqrt{2}$ so that $(\Gamma_-)^T = \Gamma_+$.

Now consider the Killing vector[2]

$$K = \bar{\epsilon}\Gamma^M\epsilon\partial_M\,. \tag{3.6}$$

Since $\Gamma^- = \Gamma^u$, it is clear that standard Killing spinors can never give rise to a Killing vector with a non-zero ∂_u-component, in agreement with the fact that generic plane waves do not have such a Killing vector. The ∂_u-component of K for an extra Killing spinor is

$$K^u = \bar{\epsilon}\Gamma^-\epsilon = \epsilon^T C\Gamma^-\epsilon\,, \tag{3.7}$$

where C is the charge conjugation matrix. C can be chosen to be Γ^0, where 0 is a frame index, and thus

$$C = \frac{1}{\sqrt{2}}(\Gamma^+ - \Gamma^-)\,. \tag{3.8}$$

Then we have

$$K^u = \frac{1}{\sqrt{2}}\epsilon^T\Gamma^+\Gamma^-\epsilon = \frac{1}{\sqrt{2}}(\Gamma^-\epsilon)^T(\Gamma^-\epsilon) \neq 0\,. \tag{3.9}$$

This shows that plane waves admitting extra Killing spinors have a Killing vector with a non-zero ∂_u-component, i.e. they are homogeneous [10].

In [10] it was shown that there are two families of homogeneous plane waves, smooth homogeneous plane waves which generalise the symmetric (Cahen-Wallach, constant A_{ij}) plane waves but are generically time-dependent, $A_{ij} = A_{ij}(u)$, and singular homogeneous plane waves which generalise the metrics with $A_{ij} \sim u^{-2}$. In Brinkmann coordinates the extra Killing vector K has the form

$$K = \partial_u + (\partial_i - \text{pieces}) \tag{3.10}$$

for smooth homogeneous plane waves, and the form

$$K = u\partial_u - v\partial_v + (\partial_i - \text{pieces}) \tag{3.11}$$

in the singular case. Thus for singular plane waves the Killing vector depends explicitly on v. Since it is easy to see that Killing spinors (be they standard or "extra") can never depend on v, this implies that singular plane waves can have no extra supersymmetries. In particular, the existence of extra Killing spinors implies geodesic completeness.

In summary, the existence of an extra Killing spinor implies that the plane wave is a smooth homogeneous plane wave. We will present the Killing spinor equations for such metrics in the next section.

[2] For a recent systematic discussion of bispinors of eleven-dimensional supergravity see e.g. [27].

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3.2 The killing spinor equation for homogeneous plane waves

In [10] it was shown that the most general smooth homogeneous plane wave can be written in stationary coordinates as (cf. (2.12))

$$ds^2 = 2dudv + A_{ij}x^i x^j du^2 + 2f_{ij}x^i dx^j du + dx^i dx^i\,, \tag{3.12}$$

where A_{ij} and f_{ij} are constant symmetric and anti-symmetric matrices respectively. In the standard Brinkmann coordinates (1.1) this metric is explicitly time-dependent unless A_{ij} and f_{ij} commute. We will specialise to the particular metric that arises in the Penrose limit of the M-theory Gödel metric below.

An orthonormal frame is

$$\begin{aligned} e^+ &= dv + \frac{1}{2}A_{ij}x^i x^j du + f_{ij}x^i dx^j \\ e^- &= du \\ e^i &= dx^i\,. \end{aligned} \tag{3.13}$$

The non-zero components of the spin connection are then

$$\omega^{+i} = A_{ij}x^j du + f_{ij}dx^j \quad \omega^{ij} = -f_{ij}du\,. \tag{3.14}$$

The Killing spinor equations for M-theory are

$$(\nabla_M - \Omega_M)\epsilon = 0\,, \tag{3.15}$$

where the covariant derivatives are those obtained from the above spin-connection, and the Ω_M are the contributions from the four-form field strength,

$$\Omega_M = \frac{1}{288}\left(\Gamma_M^{PQRS} - 8\delta_M^P\Gamma^{QRS}\right)F_{PQRS}\,. \tag{3.16}$$

We restrict F to be of the homogeneous plane-wave form

$$F = \frac{1}{3!}du \wedge \xi_{ijk}dx^{ijk} \tag{3.17}$$

with constant ξ_{ijk}. Then the Ω_M are

$$\begin{aligned} \Omega_v &= 0 \\ \Omega_u &= -\frac{1}{12}\Theta(\Gamma_+\Gamma_- + 1) \\ \Omega_k &= \frac{1}{24}(3\Theta\Gamma_k + \Gamma_k\Theta)\Gamma_+\,. \end{aligned} \tag{3.18}$$

To economise notation we use the definitions

$$\Theta = \frac{1}{3!}\xi_{ijk}\Gamma^{ijk} \qquad \Phi = \frac{1}{2}f_{ij}\Gamma^{ij}\,. \tag{3.19}$$

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Acting on spinors the covariant derivatives are

$$\begin{aligned}\nabla_v &= \partial_v \\ \nabla_u &= \partial_u - \frac{1}{2}\Phi - \frac{1}{2}A_{ij}x^j\Gamma_i\Gamma_+ \\ \nabla_i &= \partial_i - \frac{1}{2}f_{ji}\Gamma_j\Gamma_+ \end{aligned} \tag{3.20}$$

and the Killing spinor equations become

$$\partial_v\epsilon = 0 \tag{3.21}$$

$$\partial_u\epsilon = \left(\frac{1}{2}\Phi + \frac{1}{2}A_{ij}x^j\Gamma_i\Gamma_+ - \frac{1}{12}\Theta(\Gamma_+\Gamma_- + 1)\right)\epsilon \tag{3.22}$$

$$\partial_i\epsilon = \tilde{\Omega}_i\epsilon \tag{3.23}$$

where we have introduced

$$\tilde{\Omega}_i = \Omega_i + \frac{1}{2}\Gamma_j f_{ji}\Gamma_+ \,. \tag{3.24}$$

We follow the analysis of [28, 1], as adapted to the non-maximally supersymmetric case in [6, 7]. The first equation (3.21) implies that Killing spinors are independent of v. Since $\tilde{\Omega}_i\tilde{\Omega}_j = 0$, the third equation can immediately be integrated to

$$\epsilon(u, x^i) = \left(1 + \sum x^i\tilde{\Omega}_i\right)\chi(u)\,. \tag{3.25}$$

If $\epsilon(u, x^i)$ is such that it is annihilated by Γ_+, then it is independent of the x^i, and from (3.22) one finds the 16 standard Killing spinors

$$\epsilon(u) = \mathrm{e}^{-\frac{1}{4}(\Theta - 2\Phi)u}\epsilon_0\,, \qquad \Gamma_+\epsilon_0 = 0 \tag{3.26}$$

(where ϵ_0 is a constant spinor) which account for the 1/2 supersymmetry of a generic plane wave.

In the following we want to consider the possibility of extra supersymmetries for which $\Gamma_+\epsilon \neq 0$. Plugging $\epsilon(u, x^i)$ into (3.22), we find a part that is independent of x^i and a part that is linear in x^i. We therefore solve separately the two parts of this equation. The x^i-independent part gives the equation

$$\partial_u\chi(u) = \left(\frac{1}{2}\Phi - \frac{1}{12}\Theta(\Gamma_+\Gamma_- + 1)\right)\chi(u) \tag{3.27}$$

which when substituted into the remainder, removing the overall factor of x^i, gives the equations

$$\left(\frac{1}{2}A_{ik}\Gamma_i\Gamma_+ + \left[\frac{1}{2}\Phi - \frac{1}{12}\Theta(\Gamma_+\Gamma_- + 1), \tilde{\Omega}_k\right]\right)\chi = 0\,. \tag{3.28}$$

Substituting the explicit expression for $\tilde{\Omega}_k$ into this equation and multiplying by $288\Gamma_k$ (no summation over k) we find

$$\left(9\Gamma_k\Theta^2\Gamma_k + 6\Gamma_k\Theta\Gamma_k\Theta + \Theta^2 - 18\Gamma_k[\Phi, \Theta]\Gamma_k - 6[\Phi, \Theta] - 144(A_{ik} + f_{ij}f_{jk})\Gamma_k\Gamma_i\right)\Gamma_+\chi(u) = 0\,. \tag{3.29}$$

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Note that the combination $A_{ik} + f_{ij}f_{jk}$ that has popped up here is essentially the Riemann tensor of the homogeneous plane wave. We see that this is an equation for $\Gamma_+\chi(u)$ only. Solving (3.27), we find

$$\Gamma_+\chi(u) = \Gamma_+ \mathrm{e}^{\frac{1}{2}u(\Phi - \frac{1}{6}\Theta(\Gamma_+\Gamma_- + 1))}\chi_0 = \mathrm{e}^{\frac{u}{2}(\Phi + \frac{1}{6}\Theta)}\Gamma_+\chi_0\,. \tag{3.30}$$

At this point it is convenient to switch to $SO(9)$ γ-matrices γ_k via

$$q\Gamma_k = \gamma_k \otimes \sigma_3 \tag{3.31}$$

with

$$\gamma^{12\ldots9} = \mathbb{I}_{16} \tag{3.32}$$

and with $\Gamma_\pm$ defined in (3.5). Then we can equivalently write the condition (3.29) for the existence of extra Killing spinors as

$$M_k\eta(u) = 0\,, \tag{3.33}$$

where

$$M_k = 9\gamma_k\theta^2\gamma_k + 6\gamma_k\theta\gamma_k\theta + \theta^2 - 18\gamma_k[\phi,\theta]\gamma_k - 6[\phi,\theta] - 144(A_{ik} + f_{ij}f_{jk})\gamma_k\gamma_i\,, \tag{3.34}$$

with

$$\theta = \frac{1}{3!}\xi_{ijk}\gamma^{ijk}\,, \qquad \phi = \frac{1}{2}f_{ij}\gamma^{ij}\,, \tag{3.35}$$

and

$$\eta(u) = \mathrm{e}^{\frac{u}{2}(\phi + \frac{1}{6}\theta)}\eta_0 \tag{3.36}$$

a 16-component spinor.

Simple algebraic manipulations show that $\sum_k M_k$ is proportional to the identity matrix times the lhs of the equation of motion

$$\mathrm{tr}(A + f^2) + \frac{1}{12}\xi_{ijk}\xi^{ijk} = 0\,. \tag{3.37}$$

Thus the existence of just one extra Killing spinor is sufficient to guarantee that the equations of motion are satisfied.

3.3 The extra killing spinors of the M-theory anti-Mach plane wave

Since the number of supersymmetries can never decrease in the Penrose limit [17], we expect the M-theory anti-Mach metric (2.34) which we obtained as the Penrose limit of the M-theory Gödel metric to possess (at least four) extra supersymmetries. We will now verify this explicitly.

For the M-theory anti-Mach metric, the non-zero components of ξ are (2.36)

$$\begin{aligned}
\xi_{129} &= -2 \\
\xi_{349} &= 2P \\
\xi_{256} &= -2(1 - P^2)^{1/2} \\
\xi_{278} &= -2(1 - P^2)^{1/2}
\end{aligned} \tag{3.38}$$

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and the non-zero components of A_{ij} and f_{ij} are (2.35)

$$\begin{aligned} A_{22} &= 4(P^2-1) \\ A_{33} &= A_{44} = -1 \\ f_{12} &= -P\,. \end{aligned} \tag{3.39}$$

Taking into account the obvious symmetries of the metric and field strength it is sufficient to consider in detail only the $k = 1, 2, 3, 5, 9$ components of (3.33). For convenience we will write $P = s = \sin\theta$ and hence also $(1-P^2)^{1/2} = c = \cos\theta$. The M_k of interest are (we do not write the identity matrix explicitly in the following)

- k=1

$$-3 + 5s^2 + 2s\gamma_{1234} + 2c\gamma_{1956} + 2c\gamma_{1978} + (1-s^2)\gamma_{5678} - 3sc(\gamma_{156} + \gamma_{178}) \tag{3.40}$$

- k=2

$$12 - 10s^2 + 2s\gamma_{1234} - 4c\gamma_{1956} - 4c\gamma_{1978} + 4(1-s^2)\gamma_{5678} + 3sc(\gamma_{156} + \gamma_{178})\,. \tag{3.41}$$

- k=3

$$3 - s^2 + 2s\gamma_{1234} - c\gamma_{1956} - c\gamma_{1978} + (1-s^2)\gamma_{5678}\,. \tag{3.42}$$

- k=5

$$-6 + 4s^2 - 2s\gamma_{1234} + 4c\gamma_{1956} - 2c\gamma_{1978} - 4(1-s^2)\gamma_{5678} + 3sc(\gamma_{156} - \gamma_{178})\,. \tag{3.43}$$

- k=9

$$3 + s^2 + 4s\gamma_{1234} - 2c\gamma_{1956} - 2c\gamma_{1978} - (1-s^2)\gamma_{5678}\,. \tag{3.44}$$

With a little algebra one can easily show that all of these expressions have the general form

$$M_k = A_k(1+\gamma_{5678}) + B_k(1 - c\gamma_{1956} + s\gamma_{1234}) + C_k(1 - c\gamma_{1978} + s\gamma_{1234})\,, \tag{3.45}$$

where the coefficients A_k, B_k, C_k can include other gamma matrices that are not important for our discussion. It is then easily checked that

$$M_k P_1 P_2 = 0 \tag{3.46}$$

for all k, where

$$\begin{aligned} P_1 &= \frac{1}{2}(1-\gamma_{5678}) \\ P_2 &= \frac{1}{2}(1 + c\gamma_{1956} - s\gamma_{1234}) \end{aligned} \tag{3.47}$$

are two commuting projection operators,

$$P_1^2 = P_1\,, \qquad P_2^2 = P_2\,, \qquad P_1 P_2 = P_2 P_1\,. \tag{3.48}$$

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Moreover, one can check that P_1P_2 commutes (!) with $\phi + \frac{1}{6}\theta$, so that the extra Killing spinors are (cf. (3.36))

$$\eta(u) = \mathrm{e}^{\frac{u}{2}(\phi + \frac{1}{6}\theta)} P_1 P_2 \eta_0 \,. \tag{3.49}$$

This gives precisely four extra Killing spinors, consistent with the 20 supersymmetries of the M-theory Gödel metric. We have thus demonstrated explicitly that time-dependent (homogeneous) plane waves can admit extra supersymmetries.

For $s = 0$, i.e. $P = 0$, one reproduces the result of [7] that the M-theory lift of the maximally supersymmetric five-dimensional plane wave [26] has 20 supersymmetries. In this case the projectors

$$\begin{aligned} P_1 &= \frac{1}{2}(1 - \gamma_{5678}) \\ P_2 &= \frac{1}{2}(1 + \gamma_{1956}) \end{aligned} \tag{3.50}$$

are constructed in the standard way from commuting four-vectors $\gamma^{(4)}$ of the Clifford algbra.[3]

Something interesting happens when we switch on $s = P$ or f_{ij}, i.e. when we switch on a time-dependence of the homogeneous plane wave (in Brinkmann coordinates). It is clear from (3.38) that this has the effect of switching on a component in θ, namely $2P\gamma_{349}$, which does not commute with all the other components. As a consequence the corresponding projectors cannot be built anymore from commuting elements $\gamma^{(4)}$ of the Clifford algebra alone - and indeed for P_2 to be a projector for $s \neq 0$ it is essential that γ_{1956} and γ_{1234} anti-commute!

This is clearly a general feature of extra Killing spinors of non-trivial ($f_{ij} \neq 0$) homogeneous plane waves that gives the analysis a rather different flavour. In particular, while for time-independent plane waves one can diagonalise everything in sight, this is not the case in general. This evidently complicates the analysis of the general solutions of the equations (3.33), and we will leave a more detailed investigation of these and related issues for the future.

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[3] Incidentally, there are 2 values of P for which the solution (2.34), (2.36) becomes a regular CW-wave: $P = 0$, at which value the solution has 20 supersymmetries, and $P = 1$. When $P = 1$ the solution has 24 supersymmetries, the projector being given by $\frac{1}{2}(1 - \gamma_{1234})$.

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A. More on the two-parameter family of Gödel metrics

A.1 Null geodesics

In addition to the obvious Killing vectors $\partial_\phi, \partial_t, \partial_z$ the metric (2.4) has the two Killing vectors K_i, $i = 1, 2$

$$K_i = (1+r^2)^{1/2} g_i(\phi)\partial_r + \sqrt{2}\frac{\Delta}{\Omega}\frac{r}{(1+r^2)^{1/2}} g_i'(\phi)\partial_t + \frac{1+2r^2}{r(1+r^2)^{1/2}} g_i'(\phi)\partial_\phi \,, \tag{A.1}$$

where the g_i are any two linearly independent solutions of the equation

$$g''(\phi) + g(\phi) = 0 \,, \tag{A.2}$$

e.g. $g_1 = \sin\phi$, $g_2 = \cos\phi$. There are obviously (more than) enough isometries to determine the null geodesics completely. The Killing vectors $\partial_y, \partial_\phi$ and ∂_z give us the first integrals (an overdot denotes a derivative with respect to the affine parameter τ)

$$\begin{aligned} E &= \dot{t} + \sqrt{2}\frac{\Delta}{\Omega} r^2 \dot{\phi} \\ L &= \frac{\Delta}{\Omega^2}(r^2 + (1-2\Delta)r^4)\dot{\phi} - \sqrt{2}\frac{\Delta}{\Omega} r^2 \dot{t} \\ P &= \dot{z} \,. \end{aligned} \tag{A.3}$$

By a scaling of τ we can choose $E = 1$. Moreover, because of the covariance of the Penrose limit [17], without loss of generality we can choose $L = 0$ (there are enough isometries to generate geodesics with $L \neq 0$ from those with $L = 0$). Then the first two equations lead to

$$\begin{aligned} \dot{t} &= \frac{1 + (1-2\Delta)r^2}{1+r^2} \\ \dot{\phi} &= \frac{\sqrt{2}\Omega}{1+r^2} \,. \end{aligned} \tag{A.4}$$

To determine $r(\tau)$ we make use of the conserved charges F_i associated with the Killing vectors K_i, which (with $L = 0$, $E = 1$) are

$$F_i = \frac{\Delta}{\Omega^2}\frac{1}{(1+r^2)^{1/2}} g_i(\phi)\dot{r} - \sqrt{2}\frac{\Delta}{\Omega}\frac{r}{(1+r^2)^{1/2}} g_i'(\phi) \,. \tag{A.5}$$

Solving these for $\dot{r}$ and equating the resulting expressions, one finds

$$\sqrt{2}\frac{\Delta}{\Omega}\frac{r}{(1+r^2)^{1/2}} = F_2 \sin\phi - F_1 \cos\phi \,. \tag{A.6}$$

Differentiating both sides and using the expression for $\dot{\phi}$ one obtains

$$\frac{\Delta}{\Omega^2}\frac{\dot{r}}{(1+r^2)^{1/2}} = F_1 \sin\phi + F_2 \cos\phi \,. \tag{A.7}$$

Hence squaring and adding the two equations one gets

$$\frac{\Delta}{\Omega^2}\frac{\dot{r}^2}{1+r^2} = \frac{\Omega^2}{\Delta}(F_1^2 + F_2^2) - 2\Delta\frac{r^2}{1+r^2} \,. \tag{A.8}$$

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Comparing this with the null constraint

$$\frac{\Delta}{\Omega^2}\frac{\dot{r}^2}{1+r^2} = \dot{t}^2 + 2\sqrt{2}\frac{\Delta}{\Omega}r^2\dot{t}\dot{\phi} - \frac{\Delta}{\Omega^2}(r^2+(1-2\Delta)r^4)\dot{\phi}^2 - \dot{z}^2\,, \tag{A.9}$$

one finds that these two expresions are equal (for all r) provided that the single constraint

$$\frac{\Omega^2}{\Delta}(F_1^2+F_2^2) = 1-P^2 \tag{A.10}$$

is satisfied. Using this to eliminate the F_i in favour of P, we then find the solution

$$r(\tau) = \left(\frac{1-P^2}{2\Delta+P^2-1}\right)^{1/2}\sin\omega\tau\,, \tag{A.11}$$

where

$$\omega = \left(2+\frac{(P^2-1)}{\Delta}\right)^{1/2}\Omega\,. \tag{A.12}$$

In particular, for the stringy Gödel metric (2.7) one finds (either directly or by taking the limit)

$$\begin{aligned}\dot{t} &= 1-\beta^2r^2\\ \dot{\phi} &= \beta\\ r(\tau) &= (1-P^2)^{1/2}\frac{\sin\beta\tau}{\beta}\,.\end{aligned} \tag{A.13}$$

A.2 The Penrose limit of four-dimensional Gödel-like metrics

To take the Penrose limit of these Gödel metrics along any of the above geodesics, we go to adapted coordinates [13, 15, 17], i.e. we seek a change of coordinates from $x^\mu = (r, z, \phi, t)$ to (u, v, y_1, y_2) with $u = \tau$ in such a way that $g_{uv} = 1$ and $g_{uu} = g_{u1} = g_{u2} = 0$. We can e.g. choose

$$\begin{aligned}r &= r(u)\\ dz &= dy_1 + \dot{z}(u)du\\ d\phi &= dy_2 + \dot{\phi}(u)du\\ dt &= -dv + Pdy_1 + \dot{t}(u)du\,.\end{aligned} \tag{A.14}$$

The coordinate v (the only one that may require some explanation) can be found by either of the two methods employed in [17] or, more elegantly, using the Hamilton-Jacobi method advocated in [24]. In any case, one can check that this really is an adapted coordinate system, and one can thus take the Penrose limit to find the plane wave metric

$$ds^2 = 2dudv + (1-P^2)dy_1^2 - 2\sqrt{2}P\frac{\Delta}{\Omega}r(u)^2dy_1dy_2 + \frac{\Delta}{\Omega^2}(r(u)^2+(1-2\Delta)r(u)^4)dy_2^2\,. \tag{A.15}$$

At this point, the metric appears to depend on the three parameters (Δ, Ω, P). It is not difficult, but a bit tedious, to see that in fact all the dependence on Ω and Δ can be eliminated by various scalings of the coordinates (and a redefinition of P). First of all,

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by a scaling of u and a reciprocal scaling of v one can eliminate ω. Then the remaining dependence on Ω can be eliminated by a scaling of y_2. To deal with the Δ-dependence, one writes

$$r(u)^2 + (1-2\Delta)r(u)^4 = (2\Delta-1)^{-1}(\tilde{r}(u)^2 - \tilde{r}(u)^4)\,. \tag{A.16}$$

Further scalings of both coordinates can then be used to put the metric into the form

$$\begin{aligned} ds^2 &= 2dudv + (1-Q^2)dy_1^2 - 2Qr(u)^2 dy_1 dy_2 + (r(u)^2 - r(u)^4)dy_2^2 \\ r(u)^2 &= (1-Q^2)\sin^2 u\,, \end{aligned} \tag{A.17}$$

where Q is related to P by

$$Q = \sqrt{2}P\frac{\Delta^{1/2}}{(2\Delta+P^2-1)^{1/2}}\,. \tag{A.18}$$

Note that $Q=0$ ($Q=1$) iff $P=0$ ($P=1$). This is precisely the one-parameter family of plane wave metrics one also obtains from the simpler stringy Gödel metric (2.7) with $\beta=1$, and $Q\to P$. At this point the analysis proceeds as for this special case discussed in section 2.2.

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